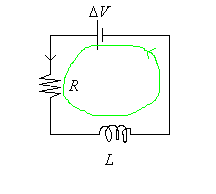
Inductors

**Problem**

Suppose we have a 10V battery connected in series to a 5H inductor and 2Ω resistor. What will be the energy stored in the magnetic field inside the inductor after a long time? If the inductor has a cross-sectional area of 3.14×10-4m2, and a length ℓ = 15cm, what is the strength of the magnetic field inside the inductor?



**Solution**

To find the current, we can use KVL, starting behind the battery.



After a long time, the current will have steadied, and will no longer be changing (and so the derivative will be 0). This will simplify our equation to:



The energy stored in the inductor will therefore be:



The magnetic field will be obtained via:

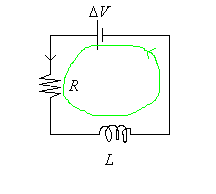


**Question 7.** MRI (magnetic resonance imaging) is a medical technique that produces detailed "pictures" of the interior of the body. The patient is placed into a solenoid that is 50 cm in diameter and 1.5 m long. A 100 A current creates a 4T magnetic field inside the solenoid. How much energy is stored inside the magnetic field, assuming the field is negligible outside the MRI machine?

Energy stored is PE = (B2/2μ0)Volume = (5)2/2(4π×10-7) ∙ (1.5)π(0.25)2 = 1.88MJ.

**Problem**

Suppose we have a 10V battery connected in series to a 5H inductor and 2Ω resistor, just like before. But suppose this time we put an iron (κm ≈ 1000) core inside the inductor. Now, what will be the energy stored in the magnetic field inside the inductor after a long time? If the inductor has a cross-sectional area of 3.14×10-4m2, and a length ℓ = 15cm, what is the strength of the magnetic field inside the inductor?



**Solution**

Given the new iron core, L will go to κmL = 5×1000H. To find the current, we can use KVL, starting behind the battery.



After a long time, the current will have steadied, and will no longer be changing (and so the derivative will be 0). This will simplify our equation to:



The energy stored in the inductor will therefore be:



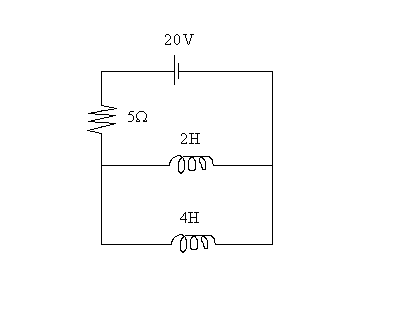
The magnetic field will be obtained via:



(k stands for ×1000, i.e., kilo)

**Problem**

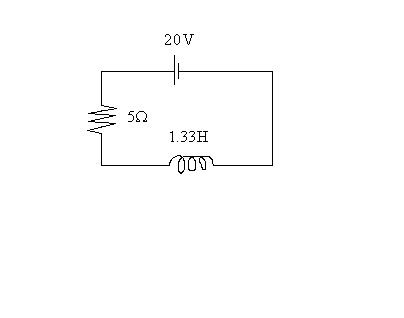
Consider the following circuit. What is the current through the resistor in the long term limit? What is the energy stored in the magnetic field in the long time limit? What is the current through the 2H, and 4H inductors in the long time limit?



**Solution**

Well, to find the current through the resitor, we can add the two inductors in parallel to get,





In the long-time limit, the current through the circuit will be steady, and thus there will be no potential drop over the inductor. And so the current will be simply,



The energy stored in the inductor will therefore be,



The current through the 2H, and 4H inductors can be obtained this way. Let i1 be going through the 2H inductor, and i2 be going through the 4H inductor. Now, from KCL



and from the energy calculation we just did,



Plugging the first equation into the second, and solving the quadratic equation yields:

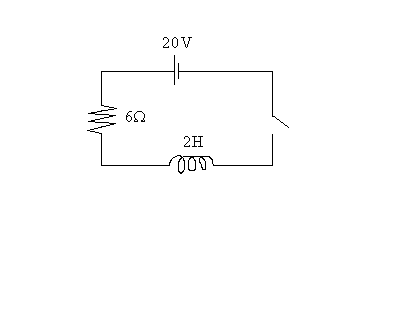


and



**Problem**

Consider the following circuit.



As soon as we flip the switch, what will be the current in the circuit? What will be current 0.1s later. What will be the steady state current?

**Solution**

Well, as soon as we flip the switch, i will be 0. The steady state current will be I = ΔV/R = 20/6 = 3.33A. The current as a function of time will be:

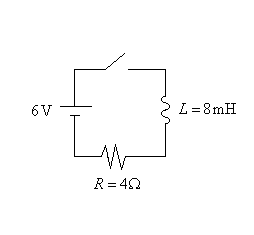


Therefore the current 0.1s later will be:



**Problem**

After flipping the switch, when will the current be equal to 1/3 its final steady state value?



**Solution**

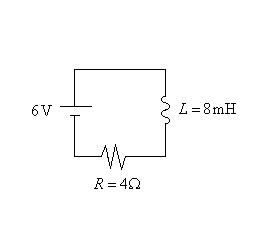
The current will build up to its steady state value, I = V/R = 6/4 = 1.5A, at an exponential pace according to the time constant τ = L/R = 8×10-3/4 = 2ms. The exact expression will be:



We want to know when I will be 1/3I­0, which is when I will be (1/3)(1.5) = 0.5A. So we set,



14. What energy is stored in the in the magnetic field set up by the inductor below?

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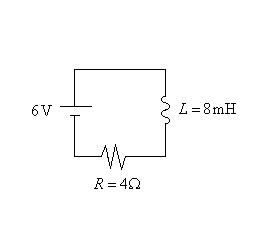


Current is given by I = ΔV/R = 6/4 = 1.5A. So,



**Problem**

What energy is stored in the in the magnetic field set up by the inductor below?

,

**Solution**

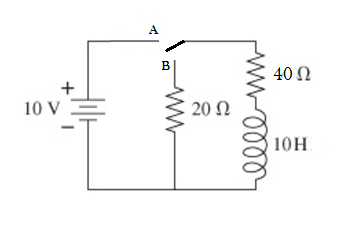
Well,



Current is given by I = ΔV/R = 6/4 = 1.5A. So,



**Question 12**. When the switch is connected to point A, what will be the current in the inductor as a function of time? When it is then connected to point B, what will be the current in the inductor as a function of time? During these processes, what is the maximum energy stored in the inductor?



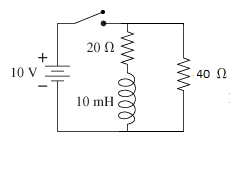
In position A, current is I(t) = I(1-e-t/τ) where I = final current = 10/40 = 0.25A, and τ = L/R = 10/40 = 0.25s. When switch is in position B, current is I(t) = Ie-t/τ, where I is initial current = 0.25A, and τ = L/R = 10/60 = 0.17s. Max energy is PE = (1/2)LImax2 = (1/2)(10)(0.25)2 = 0.31J. So

I1(t) = 0.25(1-e-4t)

I2(t) = 0.25e-6t

PE = 0.31

**Question 7.**  What is the current in the battery immediately after the switch is closed?



Immediately after, the inductor will supress current in the middle wire; current will only flow around the perimeter of the circuit. So current will be 10V/40Ω = 0.25A.